

# POSITIVE SOLUTIONS FOR A CLASS OF FRACTIONAL CHOQUARD EQUATION IN EXTERIOR DOMAIN

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ABSTRACT. This work concerns with the existence of positive solutions for the following class of fractional elliptic problems,

$$(0.1) \quad \begin{cases} (-\Delta)^s u + u = \left( \int_{\Omega} \frac{|u(y)|^p}{|x-y|^{N-\alpha}} dy \right) |u|^{p-2} u, & \text{in } \Omega \\ u = 0, & \mathbb{R}^N \setminus \Omega \end{cases}$$

where  $s \in (0, 1)$ ,  $N > 2s$ ,  $\alpha \in (0, N)$ ,  $\Omega \subset \mathbb{R}^N$  is an exterior domain with smooth boundary  $\partial\Omega \neq \emptyset$  and  $p \in (2, 2_s^*)$ . The main feature from problem (0.1) is the lack of compactness due to the unboundedness of the domain and the lack of the uniqueness of solution of the limit problem. To overcome the loss these difficulties we use splitting lemma combined with careful investigation of limit profiles of ground states of limit problem.

**Work in collaboration with professor Olimpio H. Miyagaki from Universidade Federal de São Carlos**

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